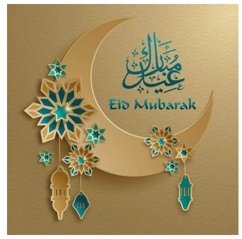
Practice Sheet 1 − Real Analysis (1)



Question One: Show that the set $\mathbb{N}$ is not bounded above.

Question Two: If $S := \{\frac{1}{n} - \frac{1}{m}\}$. Find $infS$, $supS$.

Question Three: Let S be nonempty set in $\mathbb{R}$. Let $b < 0$ and let $bS = \{bs : s \in S\}$. Prove that $inf(bS) = bsupS$ and $sup(bS) = binf(S)$.

Question four: Use the definition of the limit of a sequence to establish the following limits

1. $\lim \left(\dfrac{2n}{n+1}\right) = 2$
2. $\lim \left(\dfrac{3n+1}{2n+5}\right) = \dfrac{3}{2}$

Question five: Show that if $x_n \geq 0$ for all $n \in \mathbb{N}$ and $\lim\ x_n = 0$, then $\lim\ (\sqrt{x_n}) = 0$

Question six: Let $x_1 \geq 2$ and $x_{n+1} := 1 + \sqrt{x_n - 1}$. Shoe that (x_n) is decreasing and bounded below by 2. Find the limit.

Question seven: Prove or disprove the following statements:

1. A convergent sequence of real numbers is bounded.
2. The sum of two divergent sequences diverges.
3. Let x_n be a real sequence of real numbers. If $x_n^2 \to -x^2$ for $x \in \mathbb{R}$ for $x \in \mathbb{R}$. Then $x = 0$.
4. Let x_n be a real sequence of real numbers. If $x_n \to x$ foe $x_n > 0$. Then $x > 0$.
5. Every Cauchy sequence is a bounded sequence.
6. The sequence $x_n = \sqrt{n+1}$is a properly divergent sequence.